Non-Neutral Decision Making in Control and Dynamic Games

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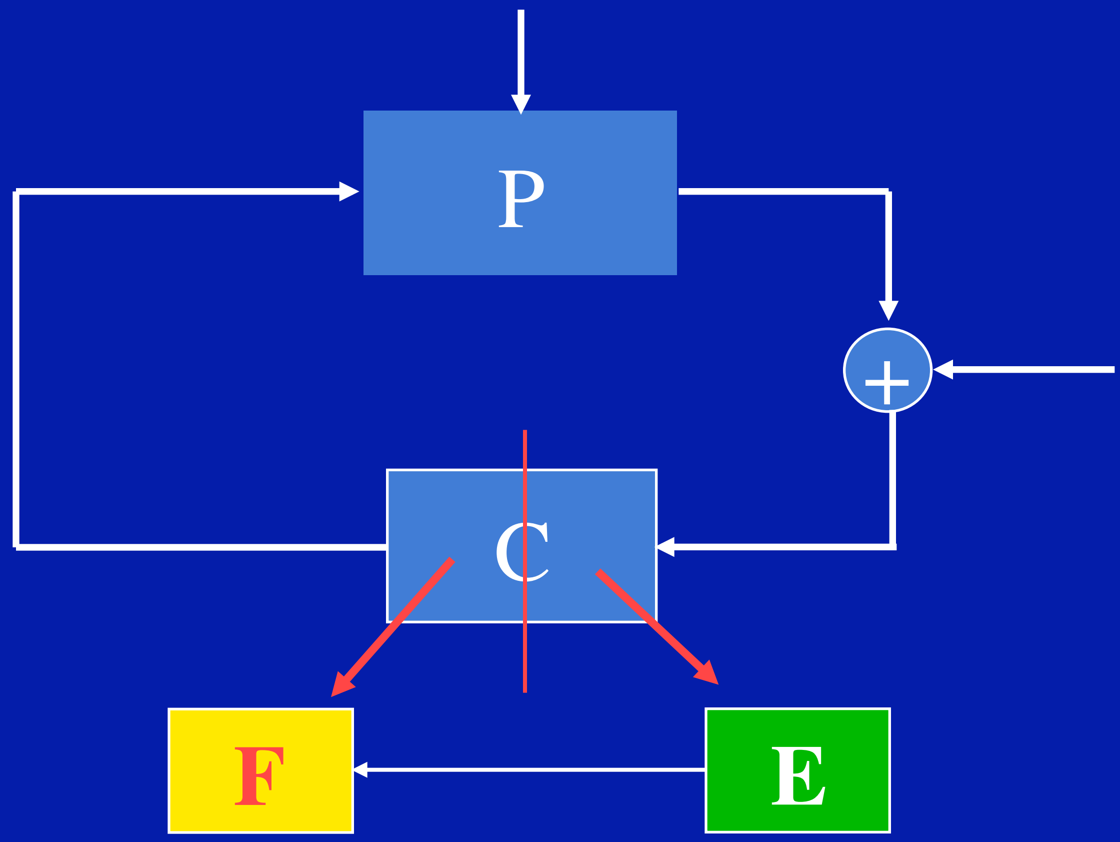
OUTLINE

- **Neutrality and non-classical information in control and dynamic games**
- **WCE and casting it in a larger class of problems**
- **Tractable problems with non-classical information**
- **Joint sensor/controller design**
- **Subtleties in games with noisy information channels (even with classical information)**
- **Conclusions**

Neutrality

A stochastic control problem is neutral if, roughly speaking, the *quality* of information carried to future stages is independent of past controls. If control policies can shape future information, then problem is non-neutral. In this case, there is generally a conflict between *action* and *probing* roles of control -- dual control.

Separation / Neutrality



A stochastic decision problem is one with *non-classical information*, if a decision unit, **B**, that *follows* another one, **A**, and *whose actions are coupled*, does not have all the information acquired and used by **A**.

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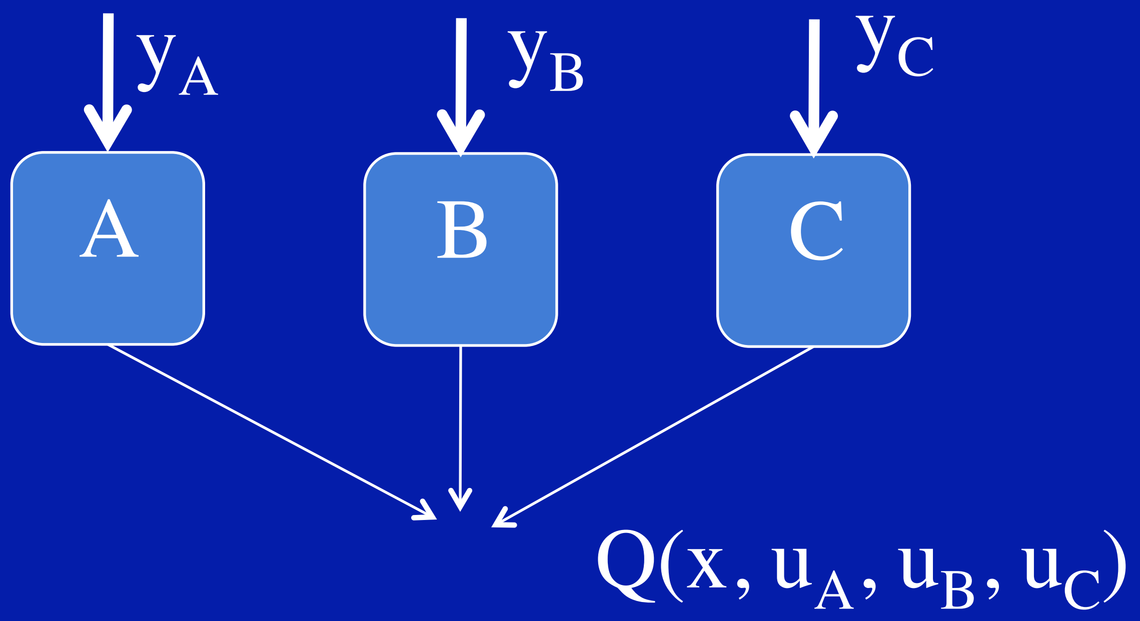
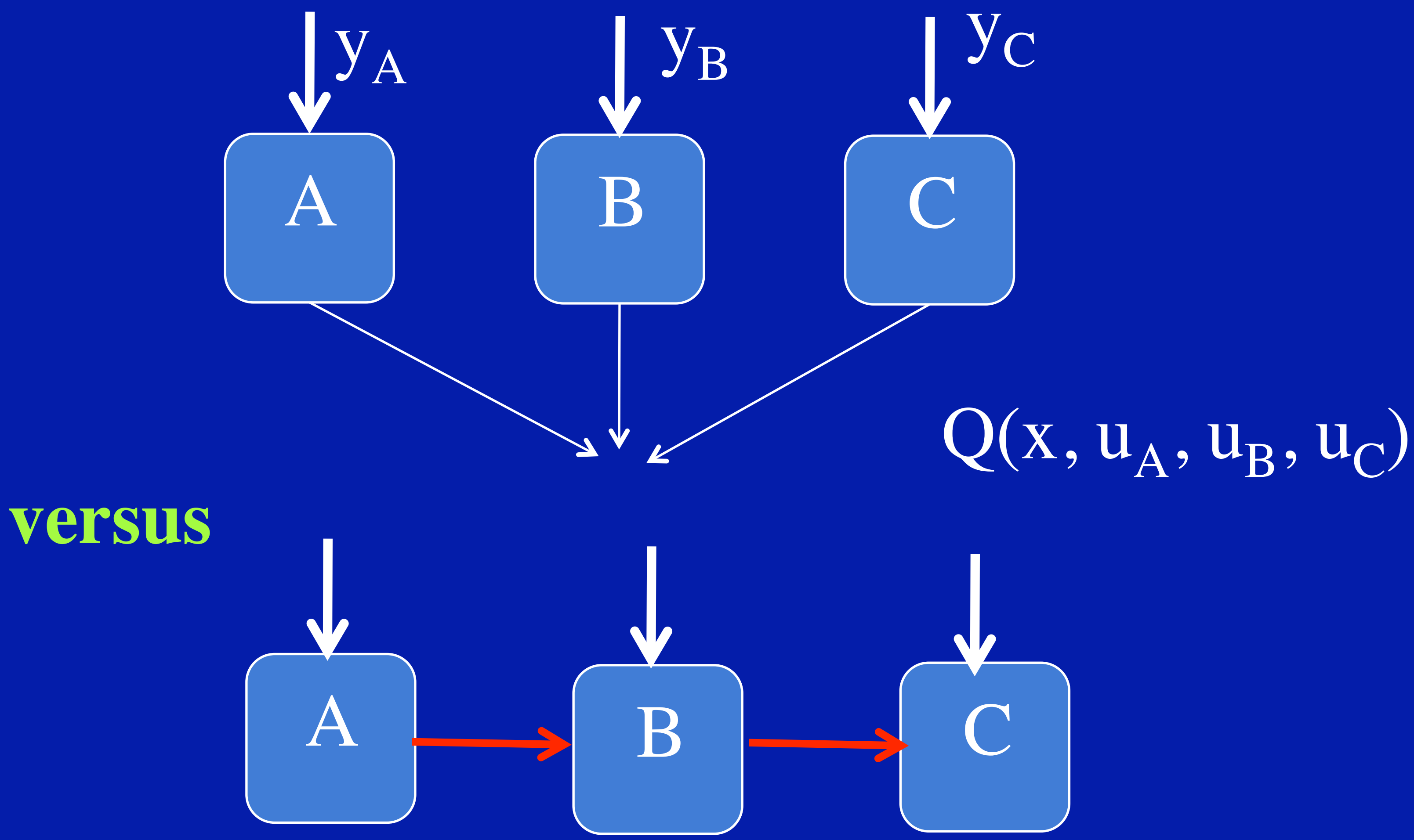
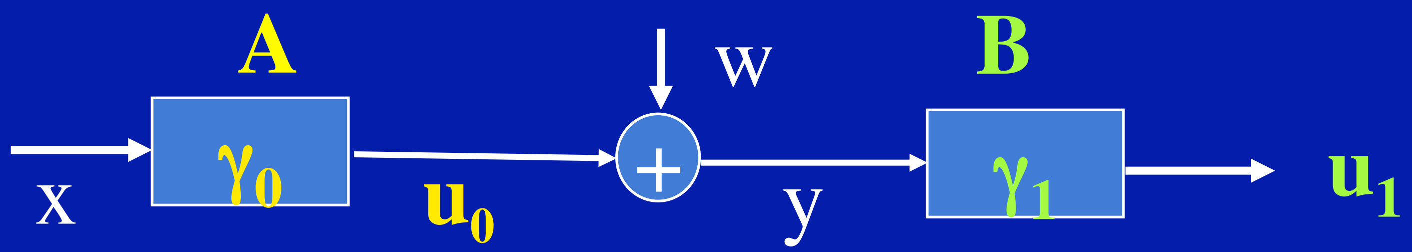


y_A
y_B
y_C
A
B
C
$Q(x, u_A, u_B, u_C)$


y_A
y_B
y_C
A
B
C
$Q(x, u_A, u_B, u_C)$
versus
A
B
C

Non-classical

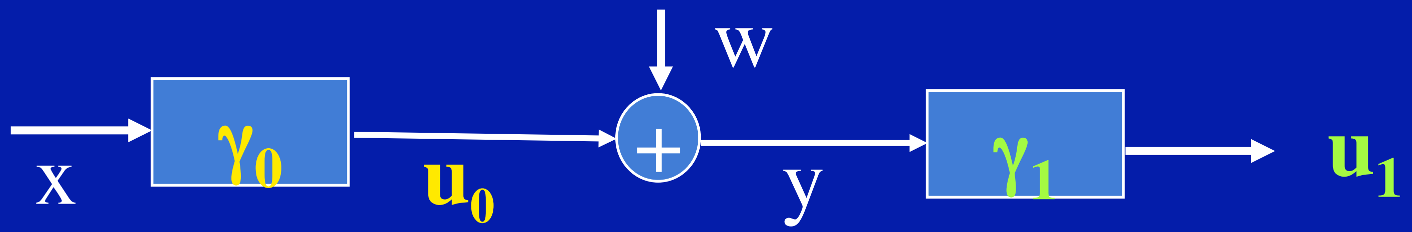


$x \sim N(0, \sigma_x^{\,2})$ $\qquad$ $w \sim N(0, \sigma_w^{\,2})$

$J(\gamma_0\ , \gamma_1) = E\ [\ Q(x, u_0, u_1)\ |\ \gamma_0\ , \gamma_1\ ]$

$J^* = \min \min J(\gamma_0\ , \gamma_1)$

Witsenhausen (1968)

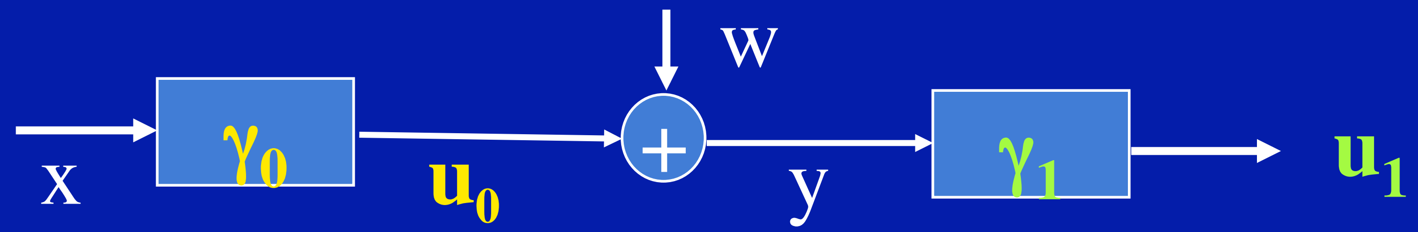


$$Q_W(x, u_0, u_1) = k_0 (u_0 - x)^2 + (u_0 - u_1)^2$$

⇒ optimal control law exists, but its structure is not known

-- roles of γ_0 and γ_1 are not aligned

Witsenhausen (1968)



$$Q_W(x, u_0, u_1) = k_0\,(u_0 - x)^2 + (u_0 - u_1)^2$$

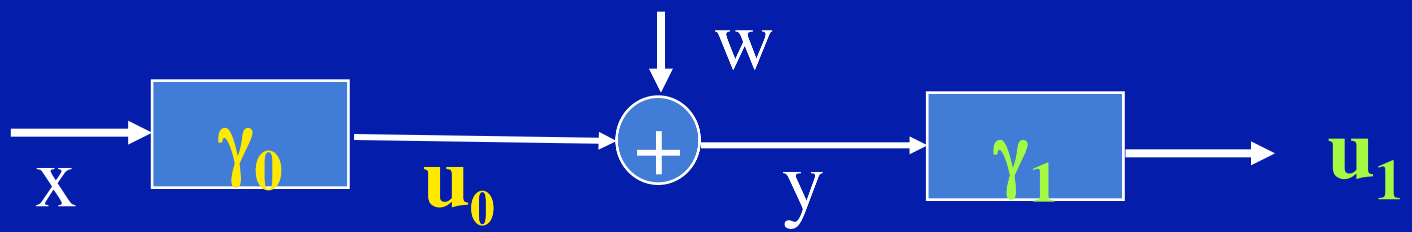
A control law that beats the best linear one:

$$\mathbf{u_0 = \gamma_0(x) = \varepsilon\ \mathrm{sgn}\,(x) + \lambda\, x}$$

$$\mathbf{u_1 = \gamma_1(y) = \mathit{E}[\varepsilon\ \mathrm{sgn}\,(x) + \lambda\, x \mid y]}$$

optimize wrt ε and λ

Gaussian Test Channel

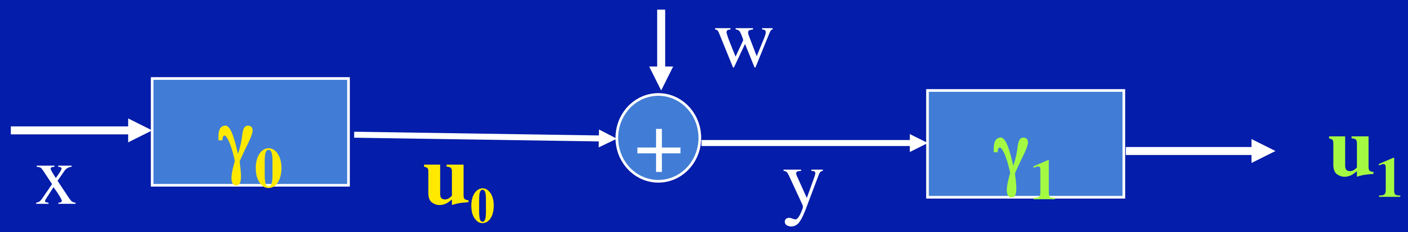


$$Q_{TC}(x, u_0, u_1) = k_0 (u_0)^2 + (u_1 - x)^2$$

optimal control law (encoder/decoder) exists, and is linear

-- roles of γ_0 and γ_1 are aligned

Generalized Gaussian Test Channel

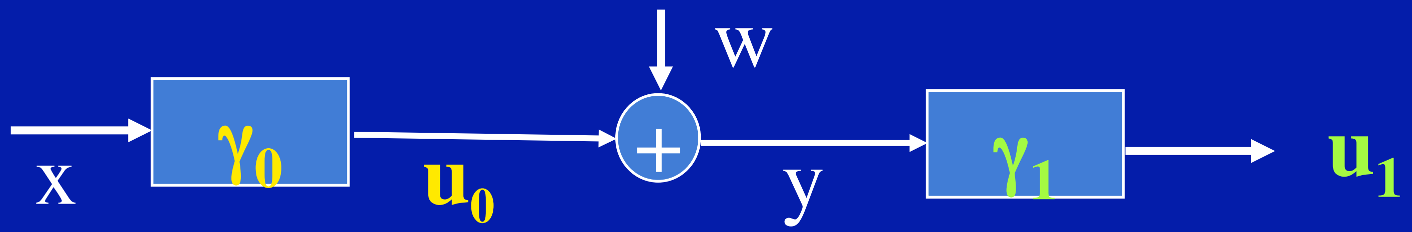


$$Q_{GTC}(x, u_0, u_1) = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

optimal control law (encoder/decoder) exists, and is linear

-- roles of γ_0 and γ_1 are aligned

Generalized Gaussian Test Channel

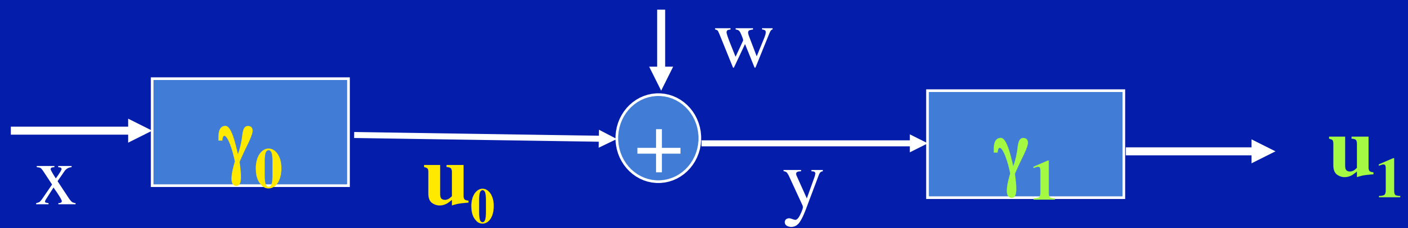


$$Q(x, u_0, u_1) = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

$$E[Q]=F(\gamma_0, \gamma_1) \geq k_0 \alpha + \beta + \inf_\gamma b_0 E[\gamma_0(x)x]$$

$$\geq k_0 \alpha + \beta - \mathrm{sgn}(b_0)\, \sigma_x \sqrt{\alpha}$$

Generalized Gaussian Test Channel



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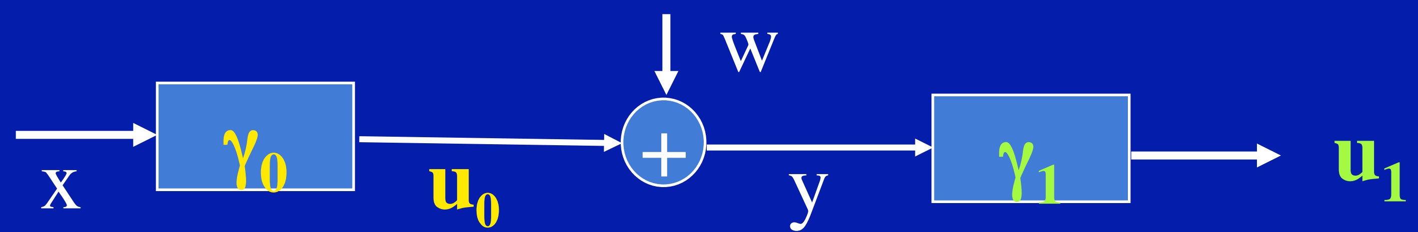
$$\geq k_0 \alpha + \beta - \mathrm{sgn}(b_0)\, \sigma_x \sqrt{\alpha}$$

DPT: $I(U_0;Y) \geq I(X;U_1)$

Generalized Gaussian Test Channel



$$Q(x, u_0, u_1) = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

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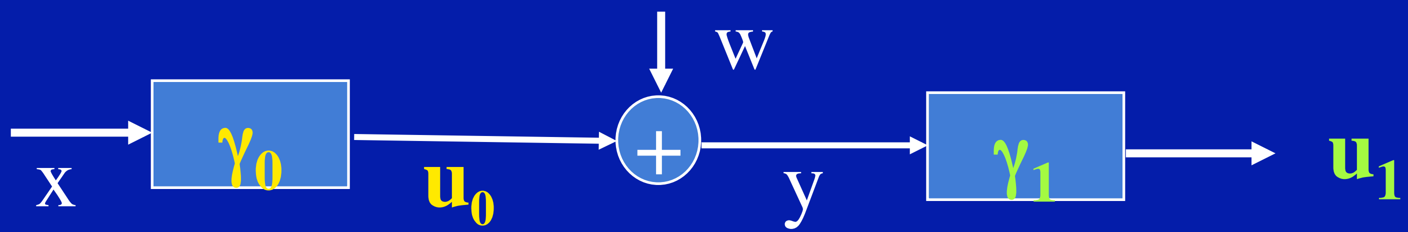
$$\geq k_0 \alpha + \beta - \mathrm{sgn}(b_0)\, \sigma_x \sqrt{\alpha}$$

$$(1/2)\log(1+(\alpha/\sigma_w^2)) \geq I(U_0;Y) \geq I(X;U_1) \geq (1/2)\log(\sigma_x^2/\beta)$$

$C(\alpha)$ $R(\beta)$

Generalized Gaussian Test Channel



$$Q(x, u_0, u_1) = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

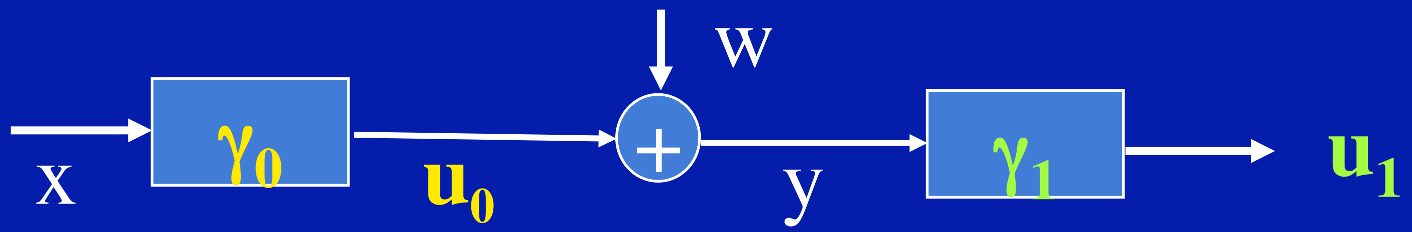
$$E[Q]=F(\gamma_0, \gamma_1) \geq k_0 \alpha + \beta + \inf_\gamma b_0 E[\gamma_0(x)x]$$

$$\geq k_0 \alpha + \beta - \mathrm{sgn}(b_0)\, \sigma_x \sqrt{\alpha}$$

$$(1/2)\log(1+(\alpha/\sigma_w^2)) \geq I(U_0;Y) \geq I(X;U_1) \geq (1/2)\log(\sigma_x^2/\beta)$$

$$\Longrightarrow \quad \beta \geq \sigma_w^2 \sigma_x^2 / (\sigma_w^2 + \alpha)$$

Generalized Gaussian Test Channel



$$Q(x, u_0, u_1) = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

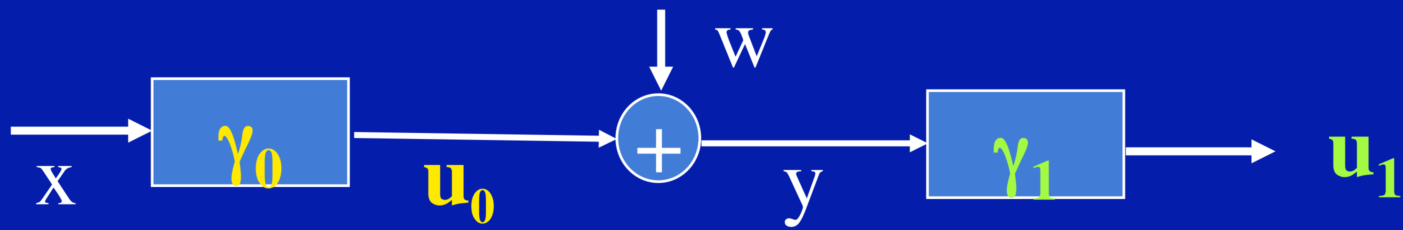
$$E[Q]=F(\gamma_0, \gamma_1) \geq k_0 \alpha + \beta + \inf_\gamma b_0 E[\gamma_0(x)x]$$

$$\geq k_0 \alpha + \beta - \text{sgn}(b_0)\, \sigma_x \sqrt{\alpha}$$

==> $\beta \geq \sigma_w^2 \sigma_x^2 / (\sigma_w^2 + \alpha)$

Inequality is tight with $\gamma_0(x) = -\text{sgn}(b_0)(\sqrt{\alpha} / \sigma_x)\, x$

Generalized Gaussian Test Channel



$$Q(x, u_0, u_1) = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

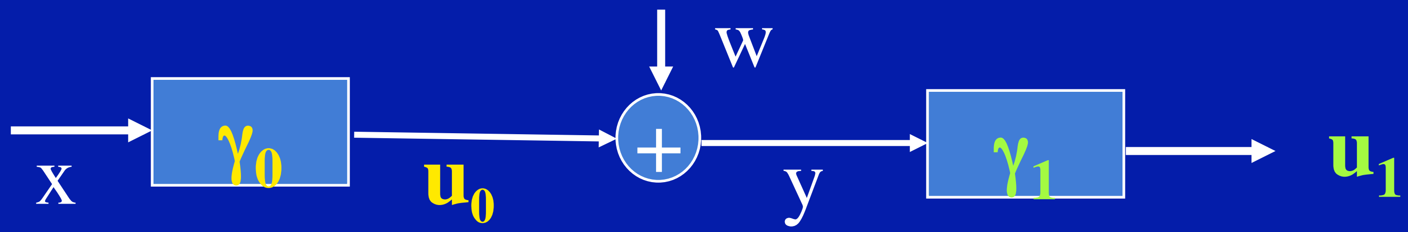
$$E[Q]=F(\gamma_0, \gamma_1) \geq k_0 \alpha + \beta - |b_0| \sigma_x \sqrt{\alpha}$$

$$\geq k_0 \alpha + \sigma_w^2 \sigma_x^2 / (\sigma_w^2 + \alpha) - |b_0| \sigma_x \sqrt{\alpha}$$

Obtain the α that minimizes the bound --> α^*

Then, $\gamma_0^*(x) = -\text{sgn}(b_0)(\sqrt{\alpha^*} / \sigma_x)\, x$, $\gamma_1^*(y) = E[x|y]$

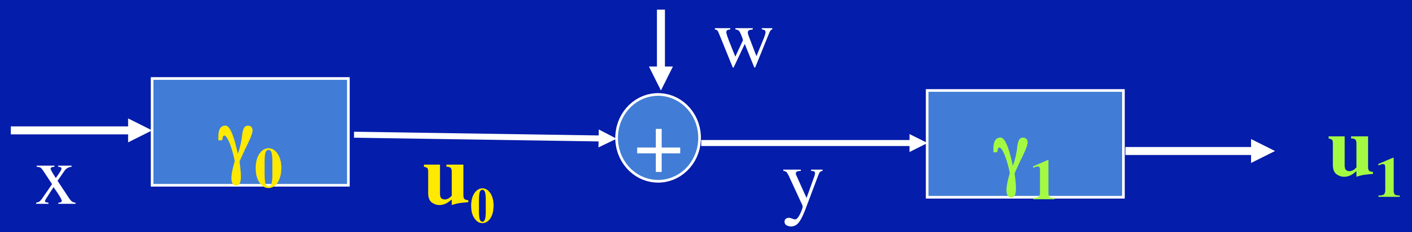
Generalized Gaussian Test Channel



$$Q(x, u_0, u_1) = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

One of the *few* instances when static/causal coding (and linear in this case) leads to attainment of equality in $C(\alpha) \geq R(\beta)$

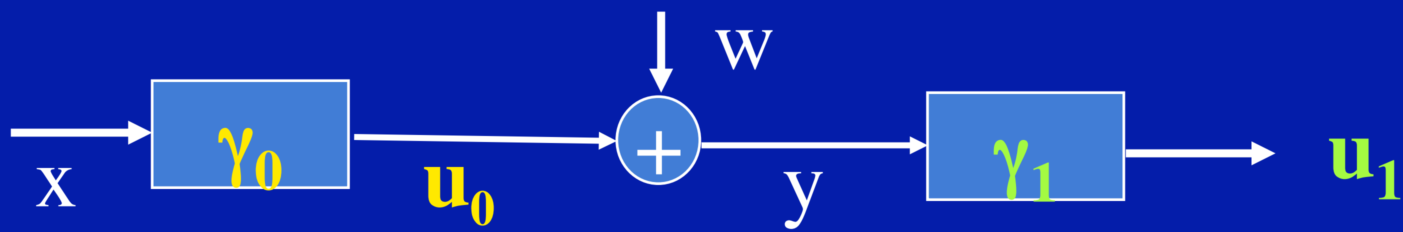
Revisit: Witsenhausen (1968)



$$Q(x, u_0, u_1) = k_0 (u_0 - x)^2 + (u_0 - u_1)^2$$

Because of the product term $u_0 \, u_1$ the preceding analysis does not apply here

However, with Conflicting Objectives

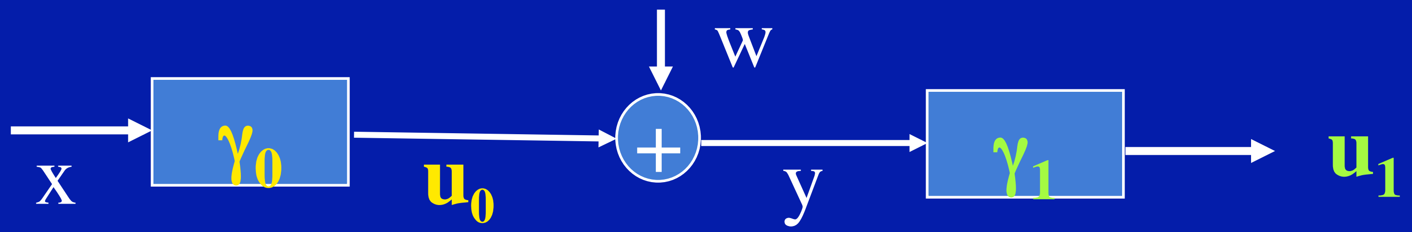


$$Q_G(x, u_0, u_1) = - k_0 (u_0 - x)^2 + (u_0 - u_1)^2$$

$$J_* = \min_{\gamma_1} \max_{\gamma_0} J(\gamma_0 , \gamma_1)$$

$\Rightarrow$ Unique saddle-point solution, control laws are linear

However, with Conflicting Objectives



$$Q_G(x, u_0, u_1) = - k_0 (u_0 - x)^2 + (u_0 - u_1)^2$$

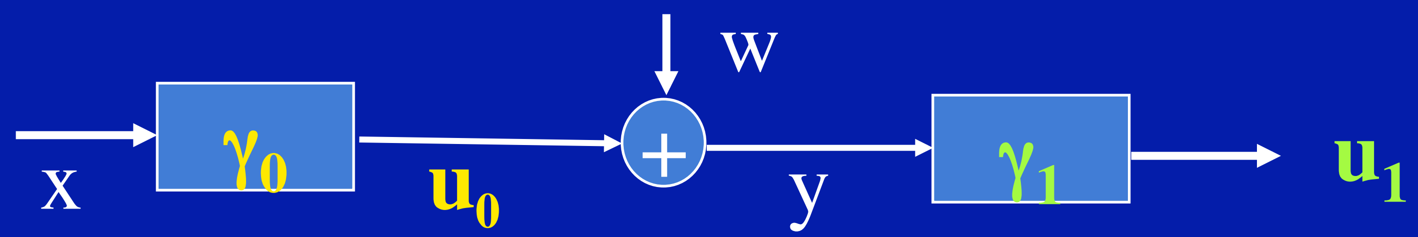
$\gamma_0^*(x) = - [k_0 / (k_0 - (\lambda^* - 1)^2)]x$, $\gamma_1^*(y) = \lambda^* y$

where λ^* **uniquely** solves the polynomial eq

$$f(\lambda) = (\sigma_w^2 / \sigma_x^2)\, \lambda[k_0 - (\lambda - 1)^2]^2 - k_0^2(1-\lambda) = 0$$

in the open interval $(\max(0, 1-\sqrt{k_0}), 1)$

Recap



$Q_W = k_0 (u_0 - x)^2 + (u_0 - u_1)^2$ conflicting roles

$Q_G = - k_0 (u_0 - x)^2 + (u_0 - u_1)^2$ aligned roles

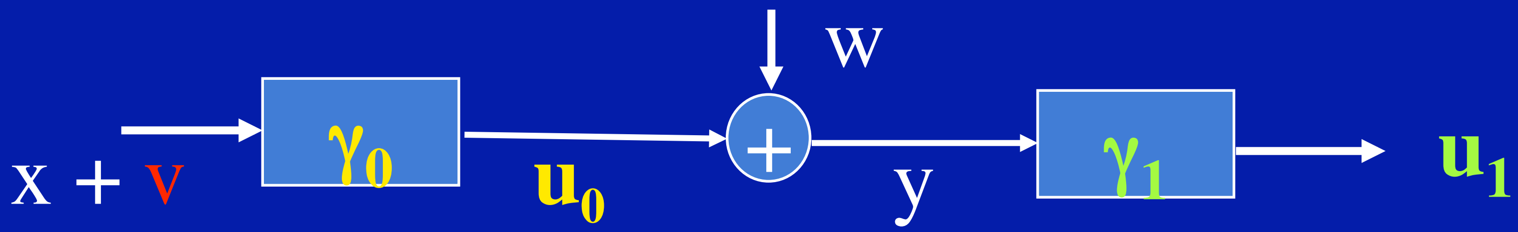
$Q_{TC} = k_0 (u_0)^2 + (u_1 - x)^2$ aligned roles

→ Not only the information structure but also the cost function is a determining factor

Extensions of the Paradigm

- Noise corrupted access to initial state
- Vector-valued variables
- Stochastic LQG teams
- Non-cooperative games
- Multi-stage joint sensor-controller design

Noise Corrupted IS

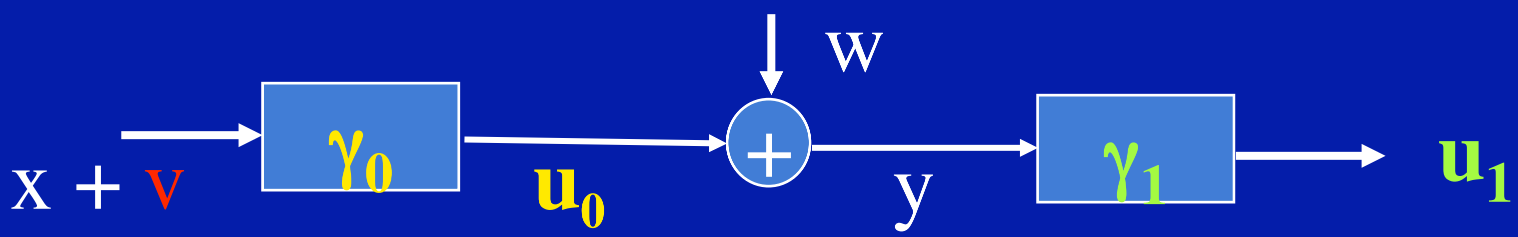


$x \sim N(0, \sigma_x^2)$, $w \sim N(0, \sigma_w^2)$, $v \sim N(0, \sigma_v^2)$

$J(\gamma_0, \gamma_1) = E[\, Q(x, u_0, u_1) \mid \gamma_0, \gamma_1 \,]$

➔ Similar structural results

Noise Corrupted IS



GTC: for some unique positive α^*

$\gamma_0^*(z) = \alpha^* \mathbf{z}$, $\gamma_1^*(\mathbf{y}) = E[\mathbf{x}|\mathbf{y}]$; $\mathbf{z := x+v}$

ZSSG: for some λ^*, root of a 5th-order polynomial

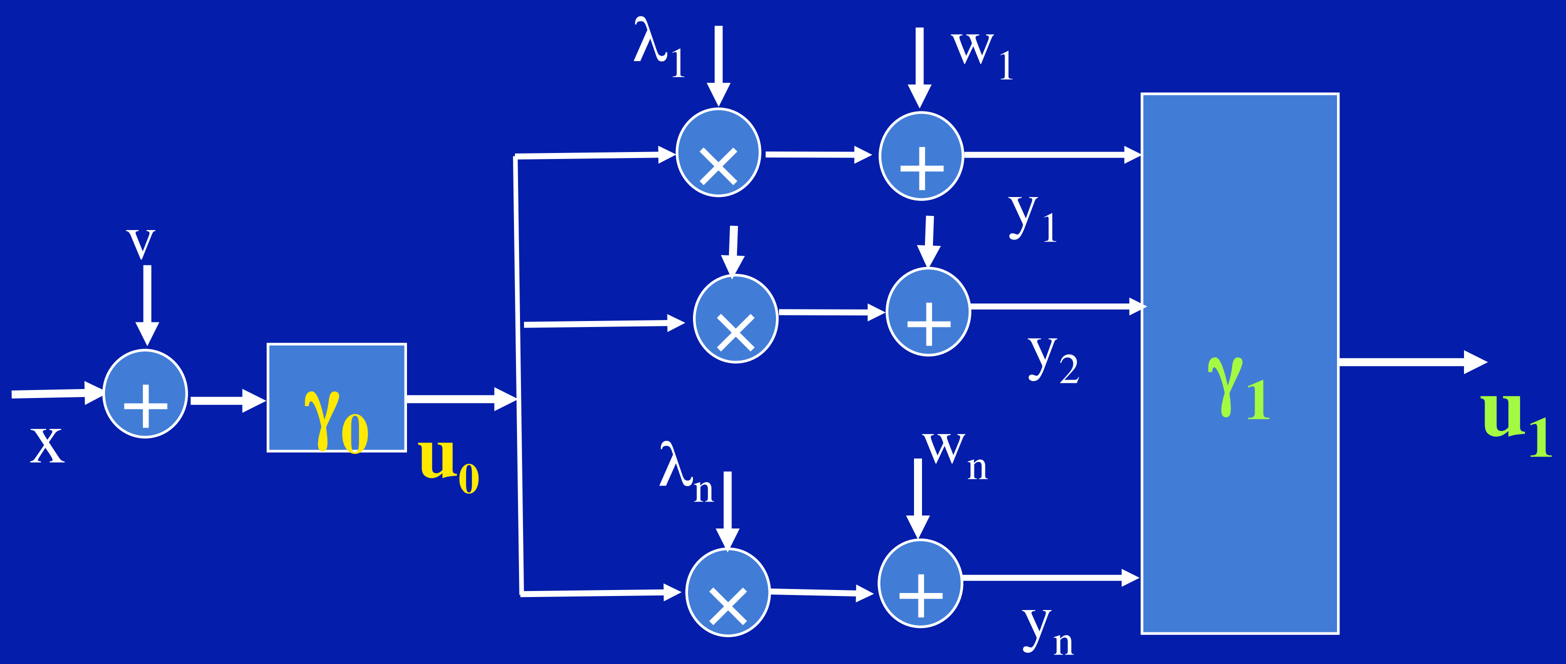
$\gamma_0^*(\mathbf{z}) = - [k_0 / (k_0 - (\lambda^* - 1)^2)][\sigma_x^2/(\sigma_x^2 + \sigma_v^2)]\mathbf{z}$

$\gamma_1^*(\mathbf{y}) = \lambda^* \mathbf{y}$

Vector-Valued Variables

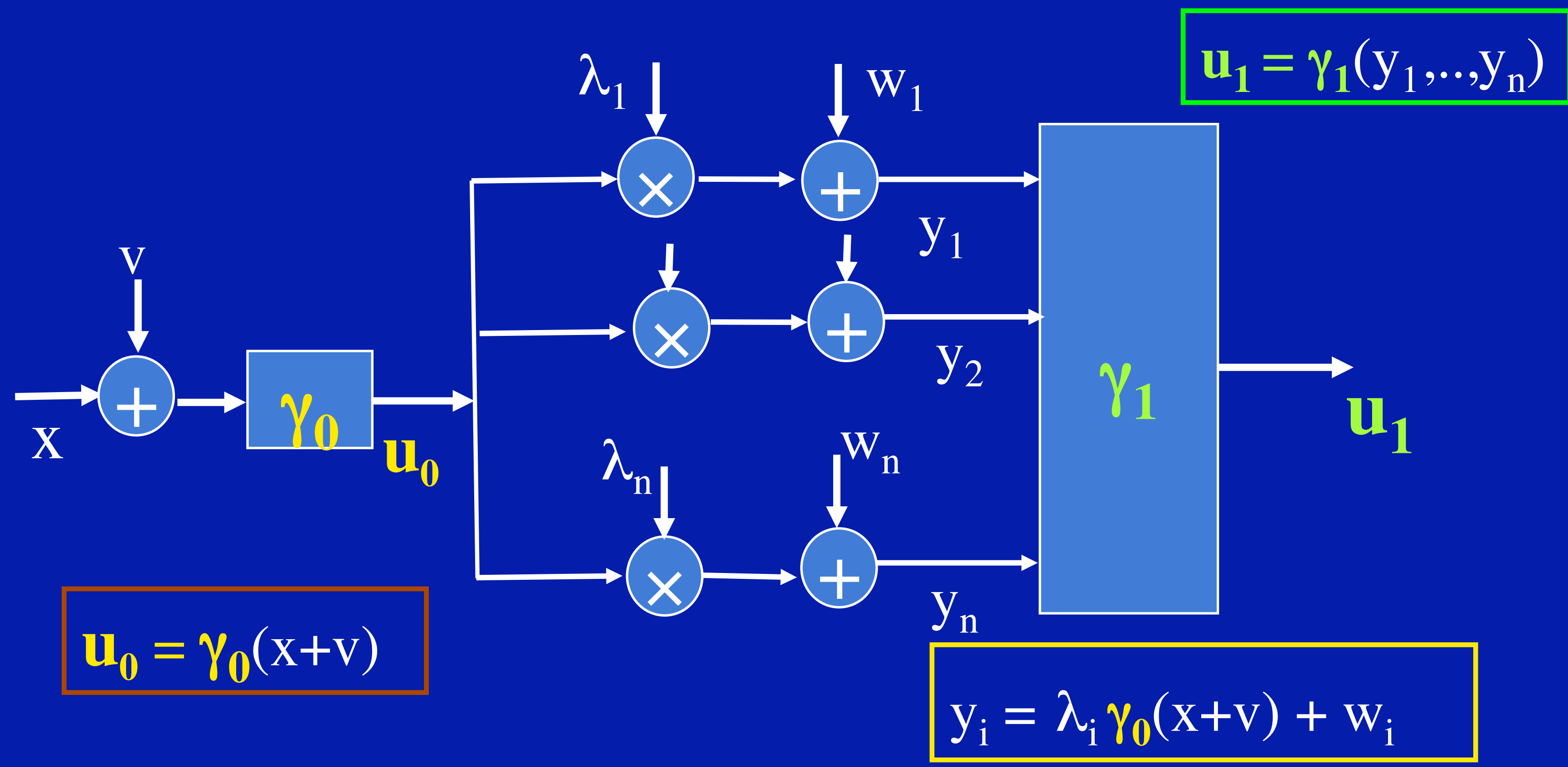
- Additional difficulties even for GTC, unless decision variables are scalar but channels are vector-valued (next)
- ZSSG is still tractable, and unique SP solution is linear

A multi-channel extension to GTC



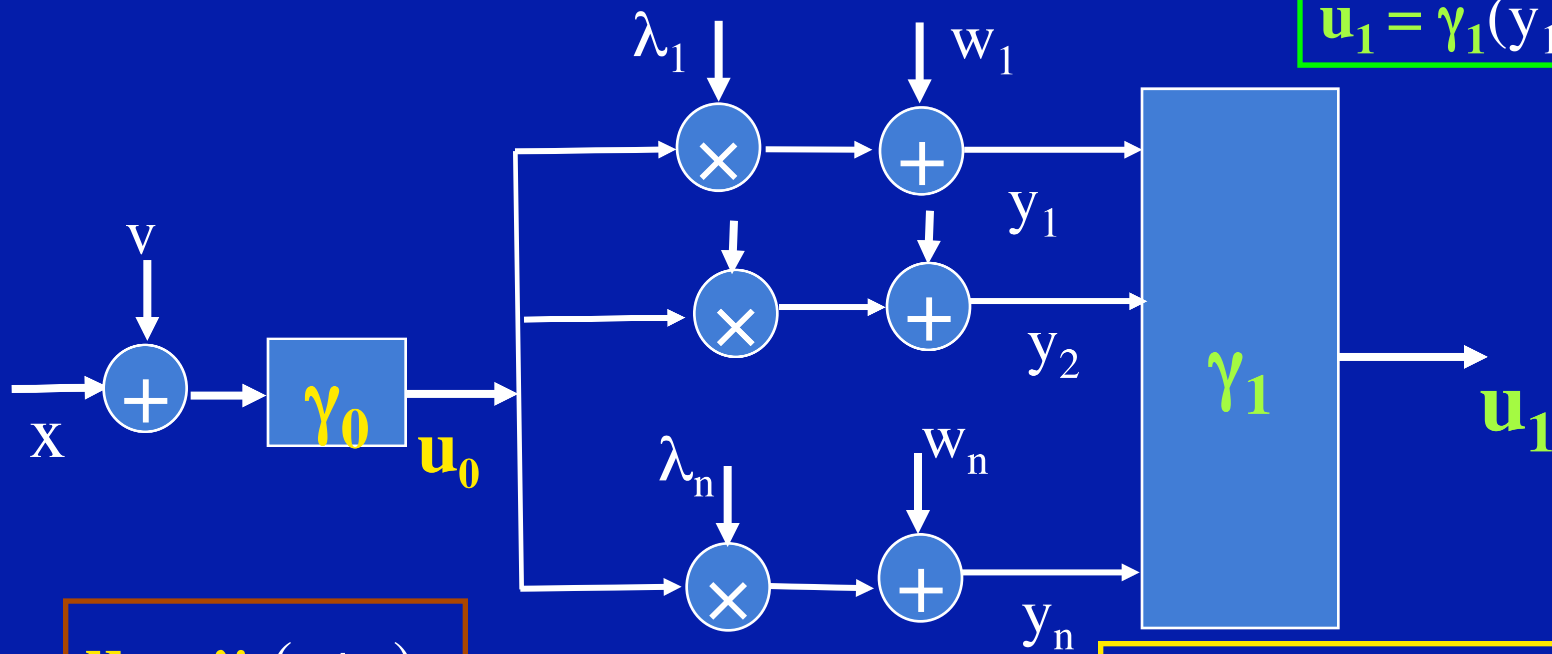
λ_i's are nonzero constants (gains);
x, v, w_i's are independent, Gaussian random variables

A multi-channel extension to GTC



A multi-channel extension to GTC

$u_1 = \gamma_1(y_1, .., y_n)$



$u_0 = \gamma_0(x+v)$

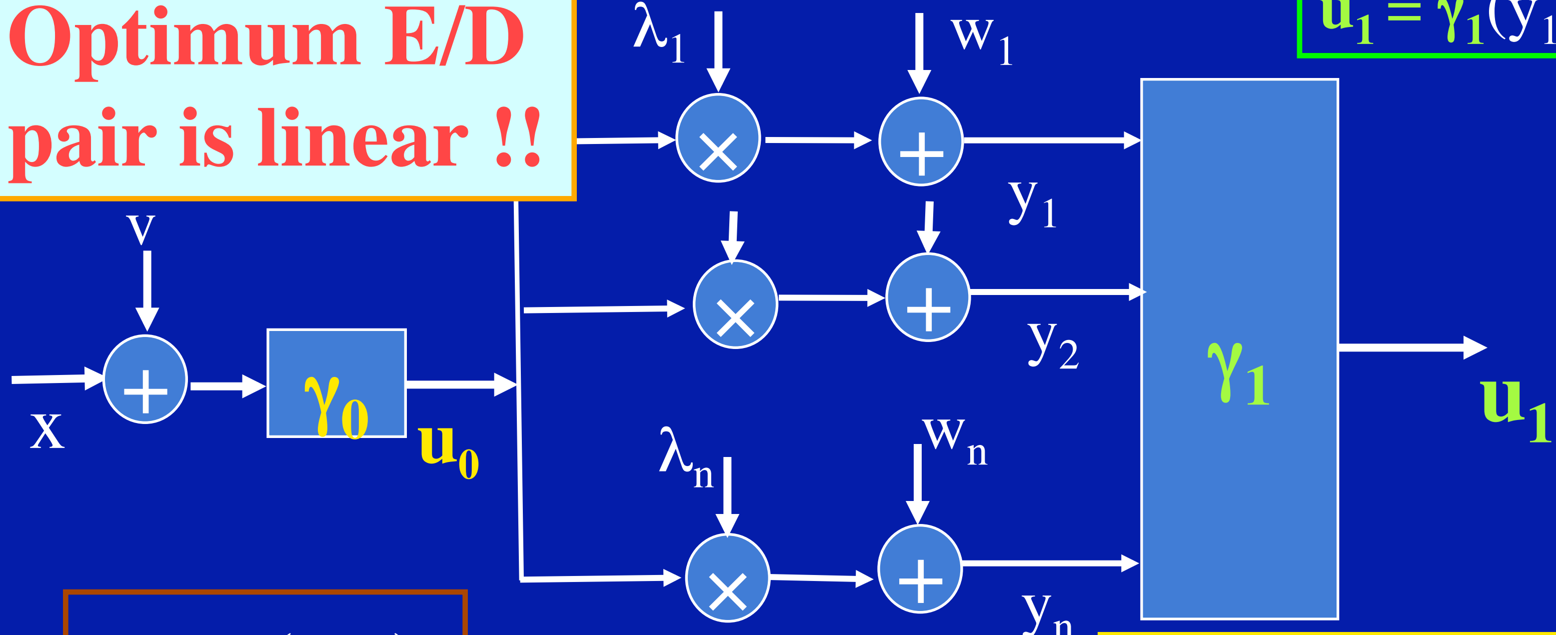
$y_i = \lambda_i \gamma_0(x+v) + w_i$

$$Q_{GTC} = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

A multi-channel extension to GTC

Optimum E/D pair is linear !!

$u_1 = \gamma_1(y_1, .., y_n)$



$u_0 = \gamma_0(x+v)$

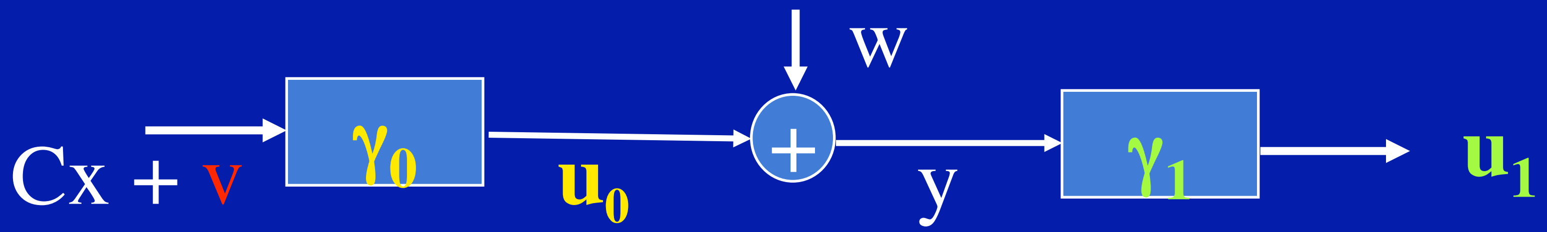
$y_i = \lambda_i \gamma_0(x+v) + w_i$

$$Q_{GTC} = k_0 (u_0)^2 + (u_1 - x)^2 + b_0 u_0 x$$

Stochastic LQG Teams

- To make tractable, one needs a *forward* channel that informs agents at the front end on garbled decentralized information received at the back end ➔ *quasi-classical*
- $\gamma_{0i}(z_i)$ at back end, i=1, …, n
- $\gamma_{1i}(y_i, z)$ at front end i=1, …, n
- For quadratic teams invoke *Radner (62) and extensions*

Vector-Valued Decision Variables (Decentralized)



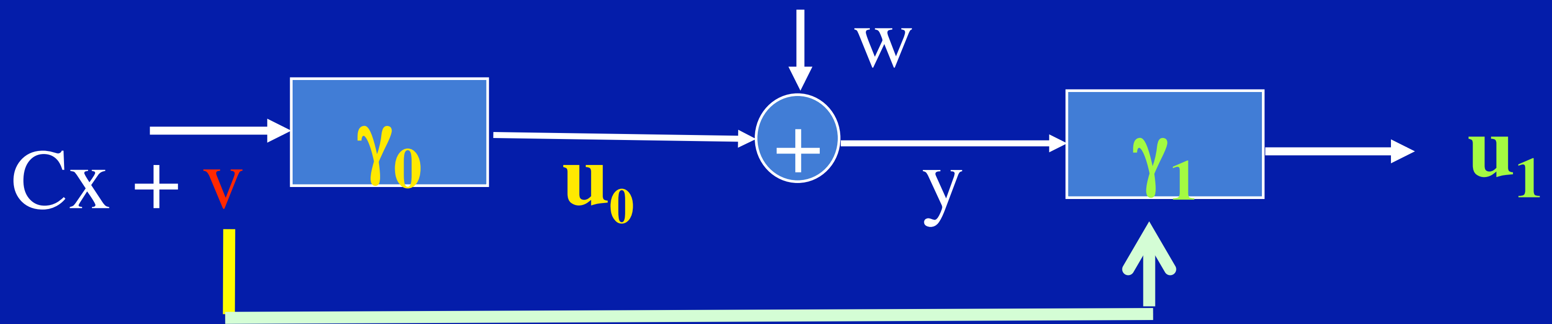
$u_{0i} = \gamma_{0i}(z_i)$, $z_i = C_i x + v_i$, $i=1, \ldots, n$

$u_{1i} = \gamma_{1i}(z, y_i)$, $y_i = D_i u_0 + w_i$, $i=1, \ldots, n$

$z = (z_1, \ldots , z_n)$; w correlated with x

$J(\gamma_0 , \gamma_1) = E\,[\, Q(x, u_0, u_1) \mid \gamma_0 , \gamma_1 \,]$

Vector-Valued Decision Variables (Decentralized)



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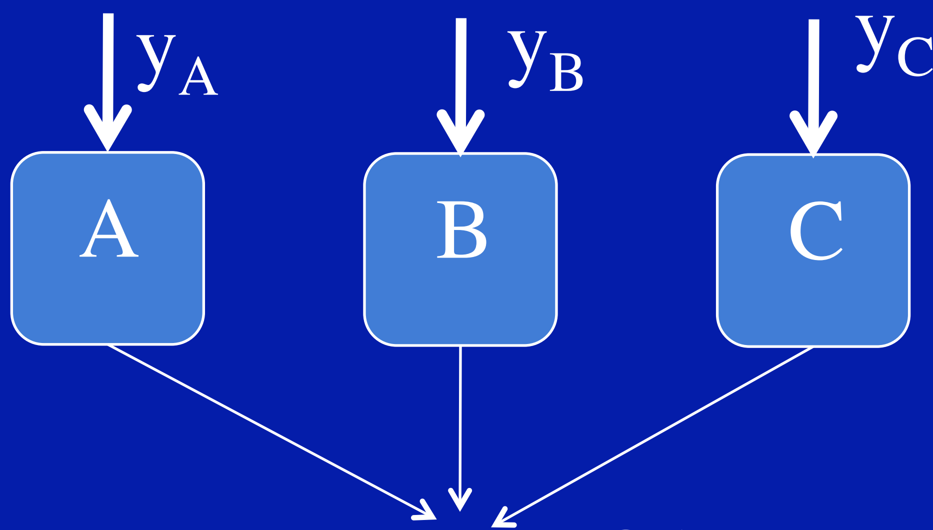
$u_{1i} = \gamma_{1i}(z, y_i)$, $y_i = D_i u_0 + w_i$,   i=1, …, n

$z = (z_1, \ldots , z_n)$; w correlated with x

$J(\gamma_0 , \gamma_1) = E\,[\, Q(x, u_0, u_1) \mid \gamma_0 , \gamma_1 \,]$



$Q(x, u_A, u_B, u_C)$

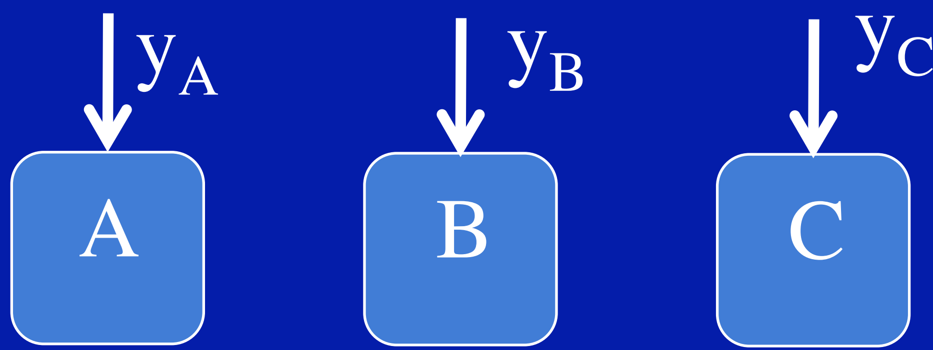
Radner (62): y's jointly Gaussian distributed, Q strictly (jointly) convex

➔ unique team optimal solution

Stochastic Nash Games

- Again one needs a *forward* channel that informs agents at the front end on garbled decentralized information received at the back end (but not actions) → *quasi-classical*
- $\gamma_{0i}(z_i)$ at back end, i=1, …, n
- $\gamma_{1i}(y_i, z)$ at front end i=1, …, n
- For quadratic games use *TB (74, 75, 78) as extension of Radner (62)*

Stochastic Nash Games



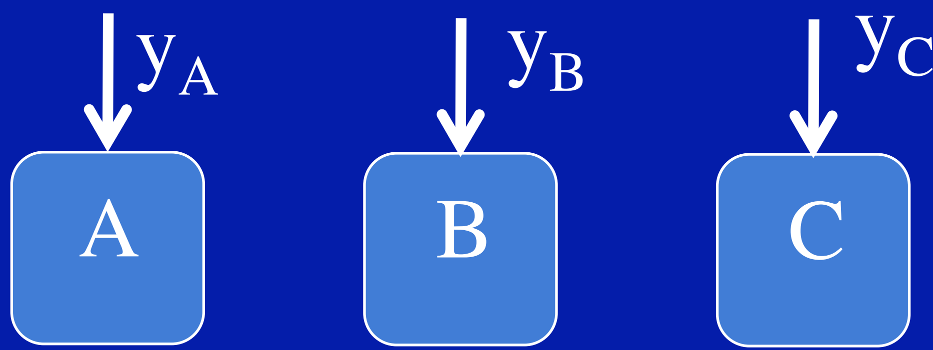
$$Q_i(x, u_A, u_B, u_C), \quad i = A, B, C$$

Nash eqm: $(\Upsilon_A, \Upsilon_B, \Upsilon_C)$

Υ_A minimizes $J_A(\Upsilon_A, \Upsilon_B, \Upsilon_C)$;

likewise for B, C

Stochastic Nash Games



$Q_i(x, u_A, u_B, u_C)$, $i = A, B, C$

TB (74, 75, 78): y's jointly Gaussian distributed, Q_i strictly convex + technical condition

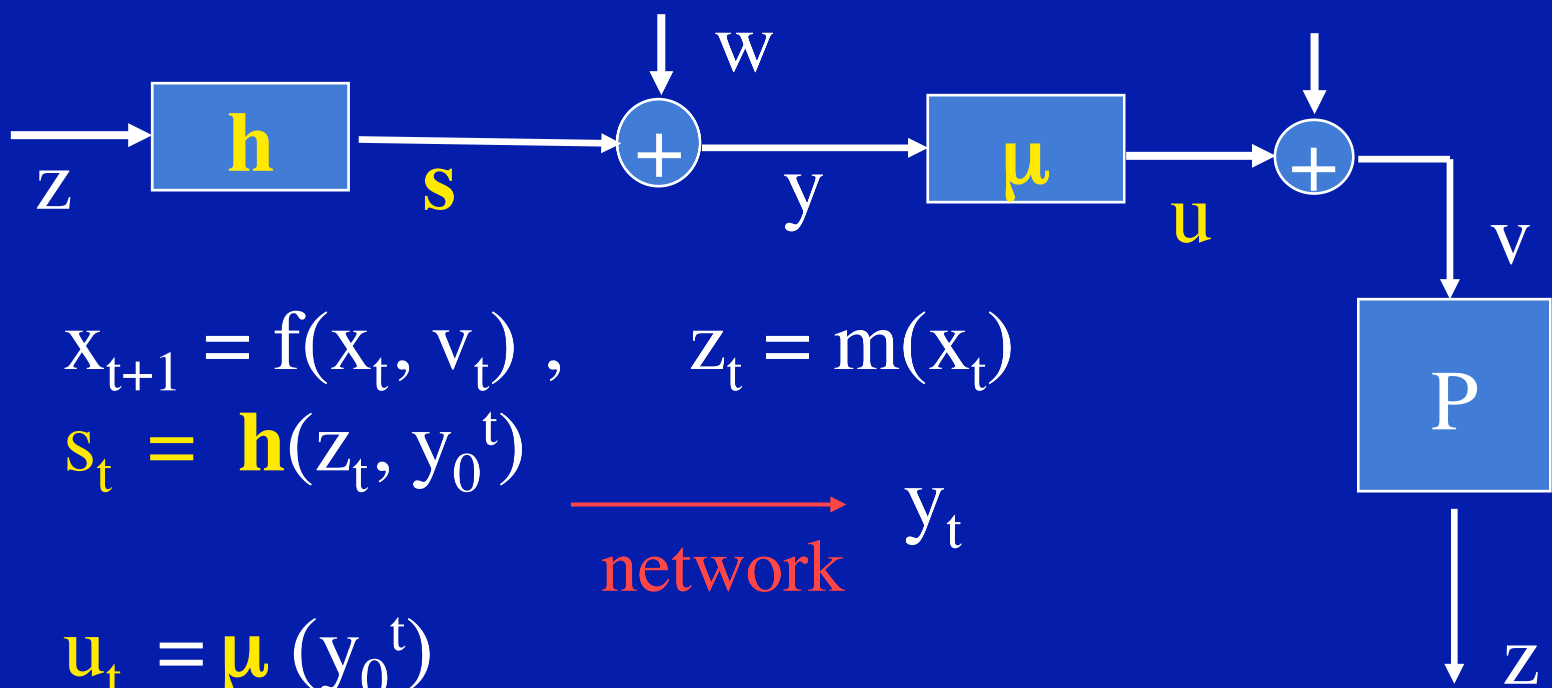
→ unique Nash eqm solution; linear

Extension to Multi-Stage Scenarios

Dynamic Systems

Joint Sensor/Controller Design

Discrete Time



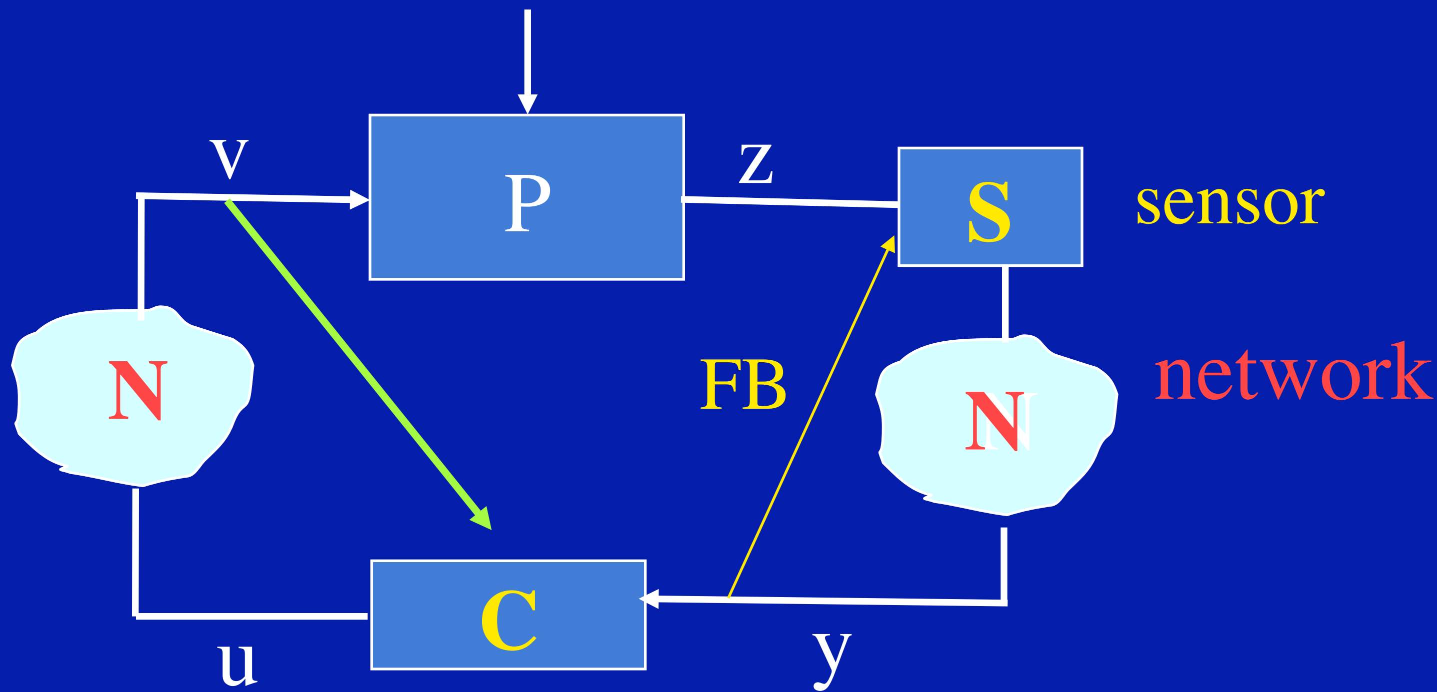
$x_{t+1} = f(x_t, v_t)$, $z_t = m(x_t)$

$s_t = \mathbf{h}(z_t, y_0^t) \xrightarrow{\text{network}} y_t$

$u_t = \boldsymbol{\mu}(y_0^t) \xrightarrow{\text{network}} v_t$

$PI = E\{\sum_t a_{t+1}(x_{t+1})^2 + b_t(u_t)^2 + q_t(s_t)^2\}$

Remote Control Paradigm



PI (S, C) $\rightarrow$ optimize

Non-classical information!

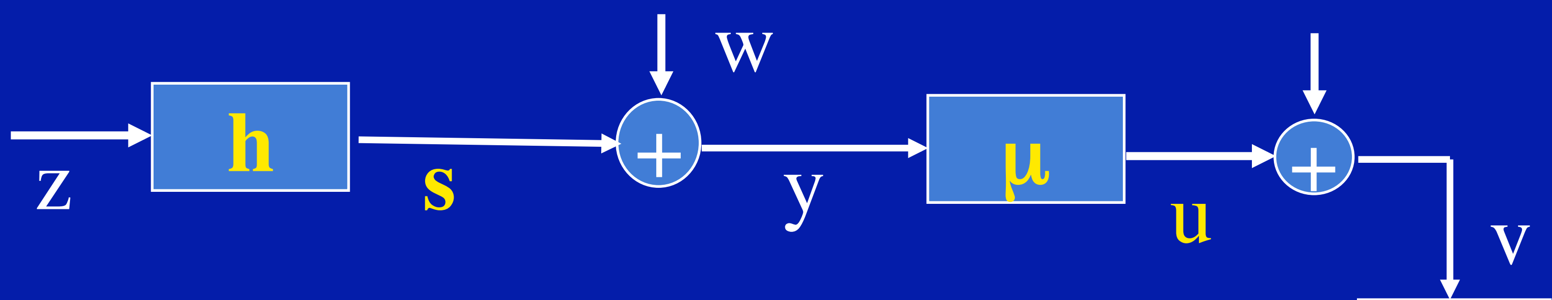
Joint Sensor/Controller Design
Discrete Time

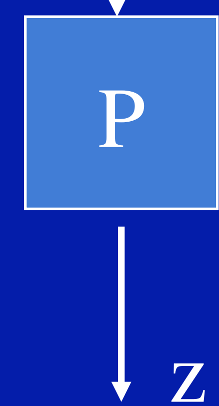
When plant is linear (scalar), and additive noises are independent Gaussian, the optimum sensor/controller pair is linear in the available information; sensor structure leads to an innovation process, independent from one stage to the next. Employs DPT recursively.*

* R. Bansal and TB, Automatica, 25(5):679-694, 1989.

Joint Sensor/Controller Design

Continuous Time





$dx_t \,/\, dt = f(x_t, v_t)$, $z_t = m(x_t)$

$s_t = \mathbf{h}(z_t, y_0^t)$ $\xrightarrow{\text{network}}$ y_t

$u_t = \boldsymbol{\mu}\,(y_0^t)$ $\xrightarrow{\text{network}}$ v_t

$PI = E\{||x||^2 + ||u||^2 + ||s||^2\}$

Details for a Special Case

$dx_t = (Ax_t + Bu_t)\, dt + F\, d\xi_t\,, \;\; t \geq 0$

$dy_t = h_t(x_t\,, y_0^t)\, dt + G\, dw_t\,, \;\; y_0 = 0$

$PI = E\{ \int_0^{tf} e^{-\beta t} [\, |x_t|_Q^2 + |u_t|^2 + |s_t|_N^2 \,]\, dt + |x_{tf}|_{Qf}^2 \}$

$u_t = \mu_t\, (y_0^t)$ $\qquad$ $s_t = h_t(x_t\,, y_0^t)$

Details for a Special Case (scalar)

$dx_t = (ax_t + bu_t)\, dt + f\, d\xi_t\,,\ \ t \geq 0$

$dy_t = h_t(x_t\,, y_0^t)\, dt + g\, dw_t\,,\ \ \ y_0 = 0$

$PI = E\{ \int_0^{tf} e^{-\beta t}\, [\ |x_t|_q{}^2 + |u_t|^2 + |s_t|_n{}^2\ ]\, dt + |x_{tf}|_{qf}{}^2$

Solution : $s_t = h_t(x_t\,, y_0^t) = H_t \cdot (x_t - \zeta_t)$

$d\zeta_t = (a\zeta_t + bu_t)\, dt + K_t\, dy_t\,,\ \ K_t = H_t\, \sigma_t / g^2$

$d\sigma_t / dt = 2a\, \sigma_t + f^2 - H_t{}^2 \sigma_t{}^2 / g^2$ IC: σ_0

$u_t = \mu_t(y_0^t) = -b\, p_t\, \zeta_t$

$dp_t / dt = -(2a - \beta)\, p_t + p_t{}^2 b^2 - q$; TC: $p_{tf} = q_f$

Solution for the Scalar Case

$s_t = h_t(x_t\ , y_0^t) = H_t \cdot (x_t - \zeta_t)$

$d\zeta_t = (a\zeta_t + bu_t)\ dt + K_t\ dy_t\ ,\ \ K_t = H_t\, \sigma_t / g^2$

$d\sigma_t / dt = 2a\ \sigma_t + f^2 - H_t^2 \sigma_t^2 / g^2$ IC: σ_0

$u_t = \mu_t(y_0^t) = -b\ p_t\ \zeta_t$

$dp_t / dt = -(2a - \beta)\ p_t + p_t^2 b^2 - q;$ TC: $p_{tf} = q_f$

⇒

$$PI = ng^2 \int_0^{tf} e^{-\beta t} [(m_t + c_t')\sigma_t]\ dt + \text{constant}$$

$c_t' := H_t^2 / g^2$ $\quad m_t := b^2 p_t^2 / ng^2$

Solution for the Scalar Case (continued)

Computation of Sensor Gain

$$\min \left\{ n g^2 \int_0^{t_f} e^{-\beta t} [m_t \sigma_t + c_t] \, dt \right\} \quad \text{wrt} \quad c_t \geq 0$$

$$d\sigma_t / dt = 2a \, \sigma_t + f^2 - c_t \sigma_t \qquad \text{IC: } \sigma_0$$

⇨ Singular control

with (possibly) multiple switches between

$c = 0$ and $c > 0$

→ optimum scheduling of sensing

Solution for the Scalar Case (continued)

Computation of Sensor Gain (2)

==> $\exists\ t_2$ such that gain is zero in $(t_2, t_f]$

==> If $\sigma_0 = 0$, $\exists\ t_1$ such that gain is zero in $[0, t_1)$

==> when not zero,

$$c_t = [\beta^2 + 2f^2 m + \sigma_t m(4a - \beta) + \sigma_t (dm/dt) - 2a\beta] / (2\sigma_t m - \beta)$$

$$\sigma_t = [\beta + \sqrt{(\beta^2 + 2f^2 m)}] / 2m$$

Solution for the Scalar Case (continued)

Numerical Example

$a=\beta=q=\sigma_0 = 0$, $b=n=g=q_f=t_f=1$ ==> $p(t) = 1/(2-t)$

Case 1: $f=1$ (control-to-plant channel not that noisy)

==> $H_t = 0 \ \forall t$ $\sigma_t = t$

Solution for the Scalar Case (continued)

Numerical Example

$a=\beta=q=\sigma_0 = 0, \quad b=n=g=q_f=t_f=1 \implies p(t) = 1/(2-t)$

Case 1: $f=1$ (control-to-plant channel not that noisy)

$\implies H_t = 0 \ \forall\ t \qquad \sigma_t = t$

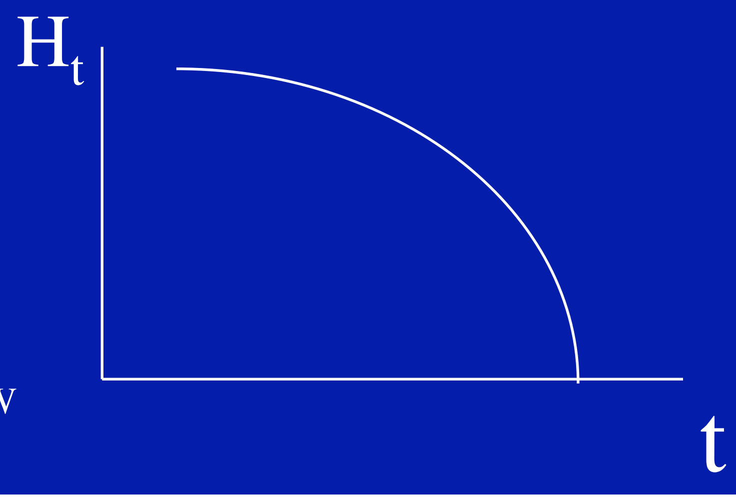
Case 2: $f=3$ (control-to-plant channel "more noisy")

switches at

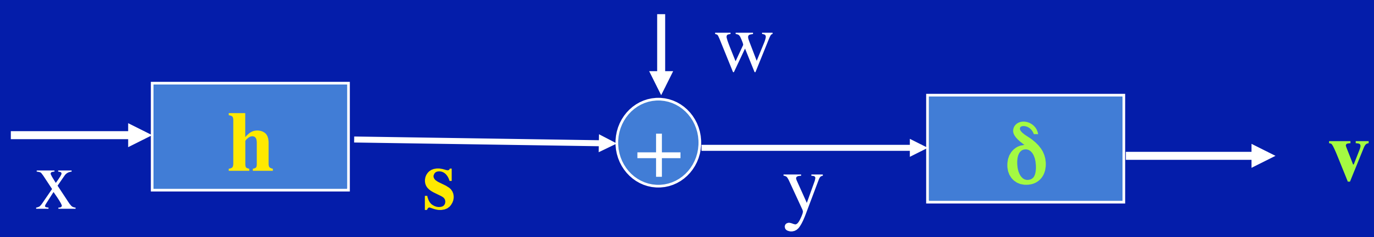
$t_1 = 0.25158$ and $t_2 = 2/3$

$H_t = \sqrt{[(9(1-t)^2 - 1) / (2-t)]}$

for $t_1 < t < t_2$



Optimality of Linear Sensor Structure

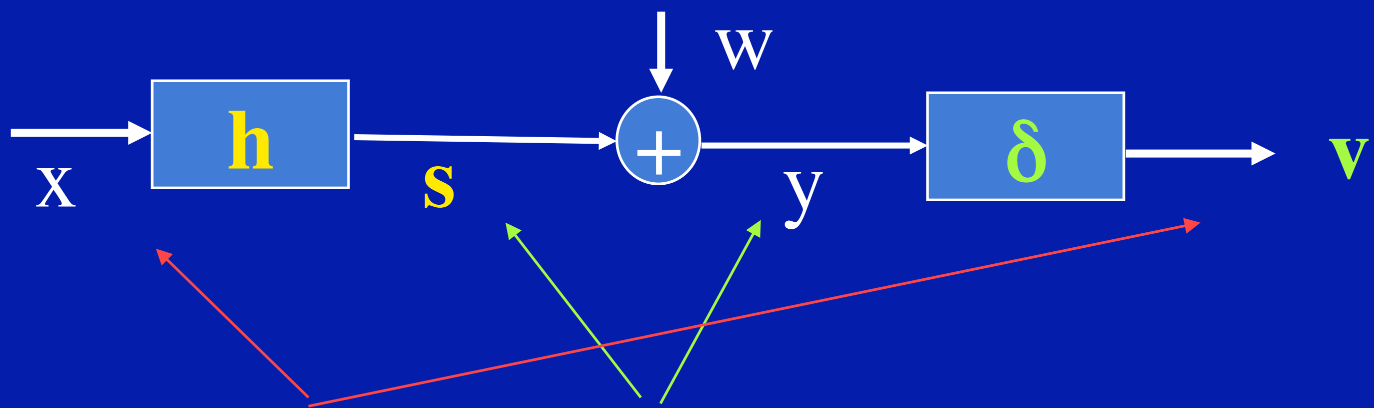


Minimize $L(h, \delta)$ over h, δ, $E\{\|h\|^2\} \leq P^2$
with feedback from y to h

$$L(h, \delta) = E\{\| v - x \|^2\}$$

$$dx_t = a\, x_t\, dt + f\, d\xi_t$$
$$dy_t = s_t\, dt + g\, dw_t$$

Optimality of Linear Sensor Structure



$$I(x\,;\,v) \leq I(s\,;\,y)$$

inversely proportional with L ==> L ≥ K(P)

bound does not change with feedback from y,

and is tight when h is linear in the innovation

Back to the Vector Case

$dx_t = (Ax_t + Bu_t)\,dt + F\,d\xi_t\,,\;\; t \geq 0$

$dy_t = h_t(x_t\,, y_0^t)\,dt + G\,dw_t\,,\;\; y_0 = 0$

$PI = E\{\int_0^{tf} e^{-\beta t}\,[\,|x_t|_Q^2 + |u_t|^2 + |s_t|_N^2\,]\,dt + |x_{tf}|_{Qf}^2\}$

$u_t = \mu_t\,(y_0^t)$ $\qquad\qquad s_t = h_t(x_t\,, y_0^t)$

Linear Solution to Vector Case

$dx_t = (Ax_t + Bu_t)\, dt + F\, d\xi_t\, ,\ \ t \geq 0$

$dy_t = h_t(x_t\, , y_0^t)\, dt + G\, dw_t\, ,\ \ \ y_0 = 0$

$PI = E\{ \int_0^{tf} e^{-\beta t} [\ |x_t|_Q^2 + |u_t|^2 + |s_t|_N^2\ ]\, dt + |x_{tf}|_{Qf}^2 \}$

Solution : $s_t = h_t(x_t\, , y_0^t) = H_t \cdot (x_t - \zeta_t)$

$d\zeta_t = (A\zeta_t + Bu_t)\, dt + K_t\, dy_t\, ,\ \ K_t = H_t\, \Sigma_t\, (GG^T)^{-1}$

$d\, \Sigma\, /dt = A\, \Sigma + \Sigma A + FF^T - H_t\, \Sigma (GG^T)^{-1}\, \Sigma\, H_t^{\ T}$

$u_t = \mu_t(y_0^t) = -B^T\, P_t\, \zeta_t$

$dP\, /\, dt = -A_\beta^{\ T}\, P - PA_\beta + PBB^TP - Q;\ \ P_{tf} = Q_f$

Optimum Sensor Gain Computation

$$\min\{ \int_0^{t_f} e^{-\beta t}\, \mathrm{Tr}[\Sigma_t M_t + \Sigma_t H_t^T N H t]\, dt \} \quad \text{wrt } \{H_t\}$$

$$M_t := P_t BB^T P_t$$

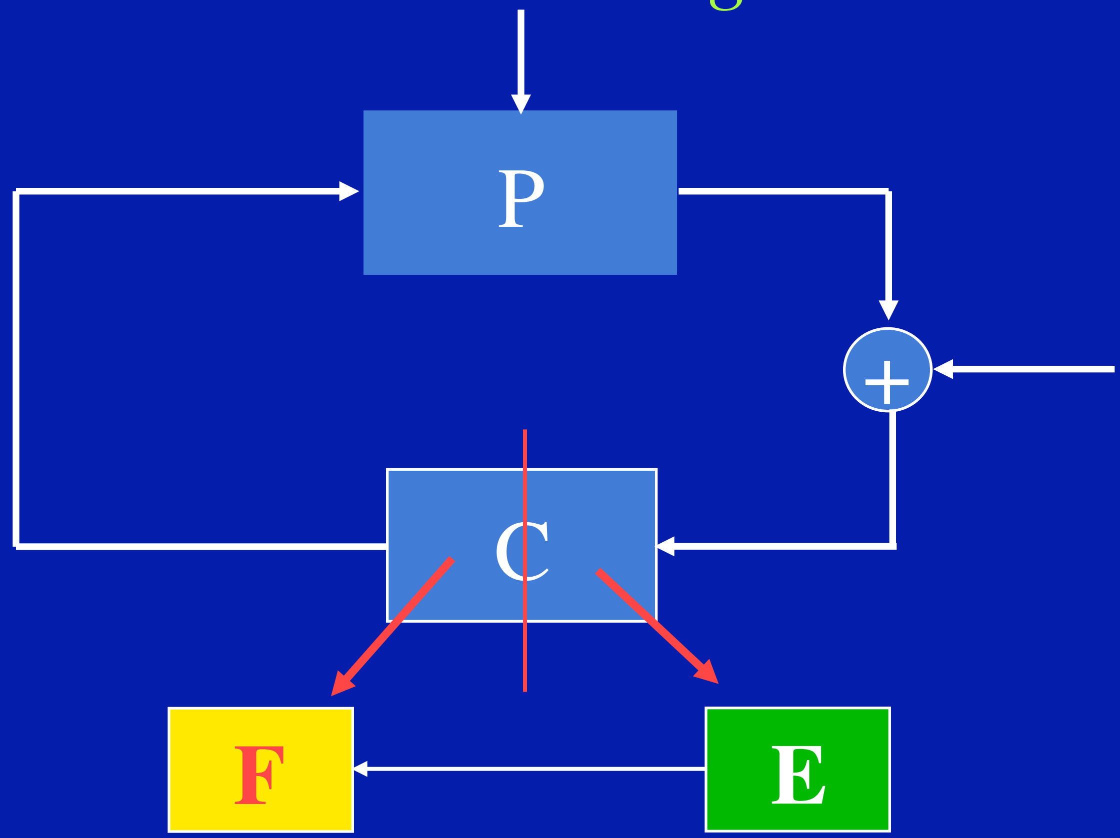
$$d\Sigma/dt = A\Sigma + \Sigma A + FF^T - H_t \Sigma (GG^T)^{-1} \Sigma H_t^T$$

-- Singular as well as impulsive controls

-- Improvement is possible with nonlinear structure

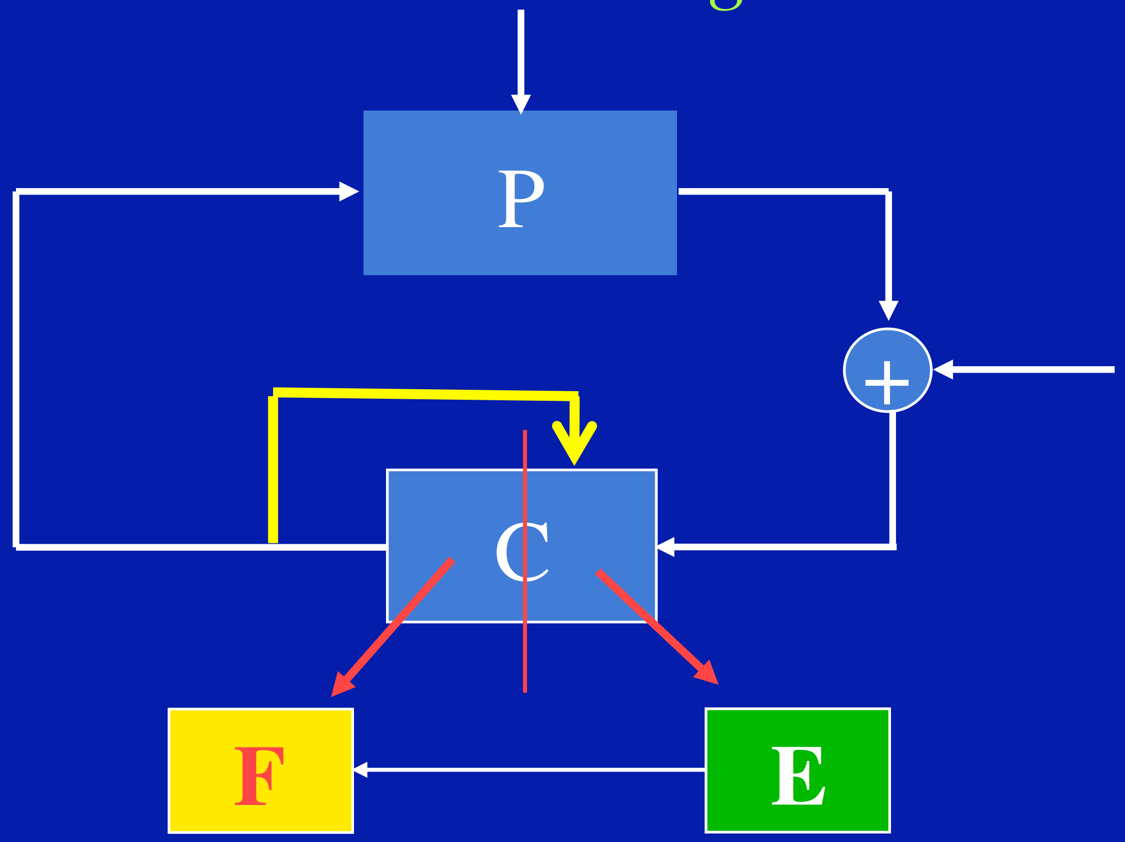
Back to Separation / Neutrality

Does it hold in games?



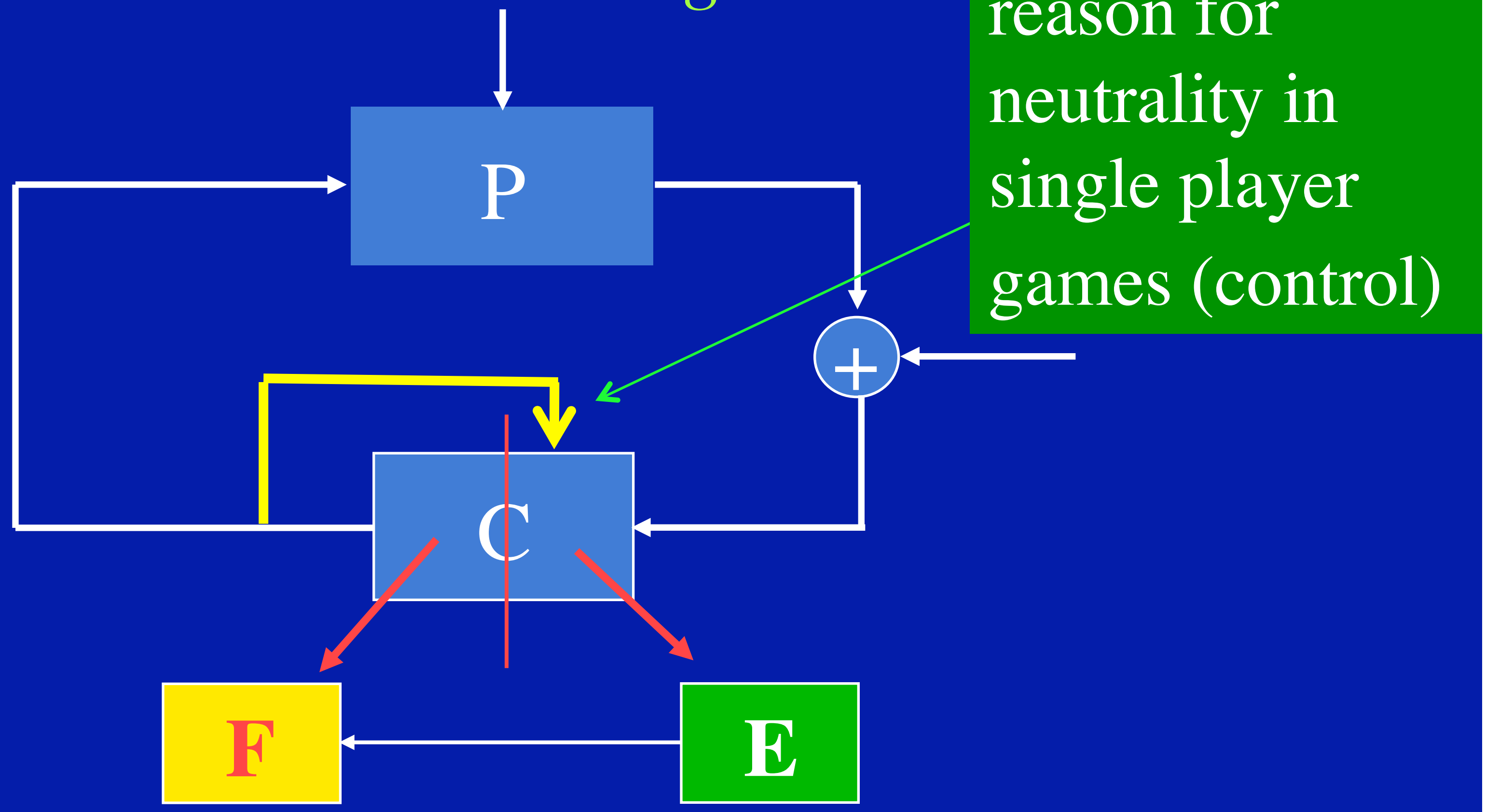
Back to Separation / Neutrality

Does it hold in games?



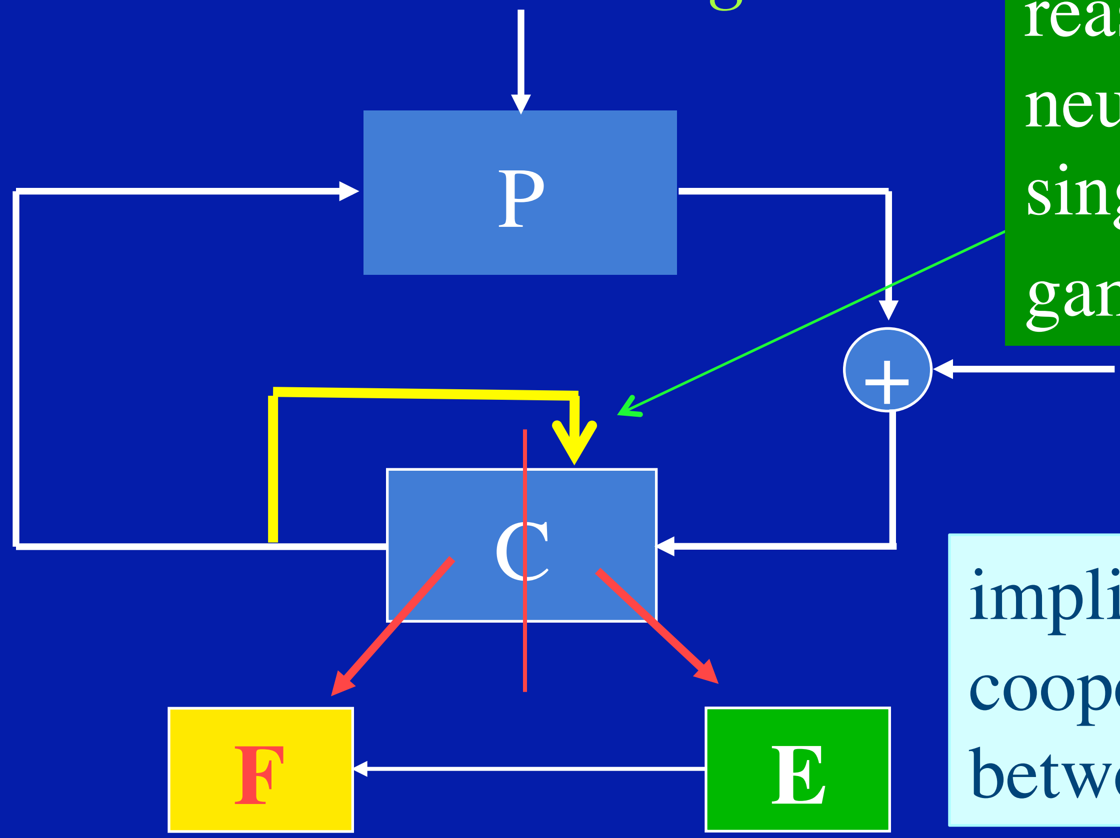
Back to Separation / Neutrality

Does it hold in games?



Back to Separation / Neutrality

Does it hold in games?



reason for neutrality in single player games (control)

implicit cooperation between F & E

ZSSDG with common measurements

$dx_t = (Ax_t + Bu_t + Dv_t)\,dt + F\,d\xi_t\,, \ \ t \geq 0$

$dy_t = Hx_t\,dt + G\,dw_t\,, \quad y_0 = 0$ (common measurement)

$u_t = \gamma_t\,(y_0{}^t) \qquad\qquad v_t = \mu_t\,(y_0{}^t)$

$PI = E\{ \int_0^{tf} [\ |x_t|_Q{}^2 + |u_t|^2 - |v_t|^2\ ]\,dt + |x_{tf}|_{Qf}{}^2 \}$

$\min_\gamma \max_\mu \ J(\gamma, \mu) \ \rightarrow \ (\gamma^*, \mu^*)$ say SP

ZSSDG with common measurements

$dx_t = (Ax_t + Bu_t + Dv_t)\, dt + F\, d\xi_t\,, \quad t \geq 0$

$dy_t = Hx_t\, dt + G\, dw_t\,, \quad y_0 = 0$ (common measurement)

$u_t = \gamma_t\, (y_0^{\,t}) \qquad\qquad v_t = \mu_t\, (y_0^{\,t})$

$PI = E\{ \int_0^{t_f} [\, |x_t|_Q^{\,2} + |u_t|^2 - |v_t|^2 \,]\, dt + |x_{tf}|_{Qf}^{\,2} \}$

$\min_\gamma \max_\mu \; J(\gamma, \mu) \;\rightarrow\; (\gamma^*, \mu^*)$

Does certainty equivalence hold?

-- can SP policies from deterministic game be used?

ZSSDG with common measurements

$dx_t = (Ax_t + Bu_t + Dv_t)\,dt + F\,d\xi_t\,, \quad t \geq 0$

$dy_t = Hx_t\,dt + G\,dw_t\,, \quad y_0 = 0$ (common measurement)

$u_t = \gamma_t\,(y_0^{\,t}) \qquad\qquad v_t = \mu_t\,(y_0^{\,t})$

$PI = E\{ \int_0^{t_f} [\ |x_t|_Q^2 + |u_t|^2 - |v_t|^2\ ]\,dt + |x_{tf}|_{Qf}^2 \}$

$\min_\gamma \max_\mu \ J(\gamma, \mu) \ \rightarrow \ (\gamma^*, \mu^*)$

Does certainty equivalence hold? *Qualified* NO

Building a common filter with u, v requires cooperation

ZSSDG with common measurements

$dx_t = (Ax_t + Bu_t + Dv_t)\, dt + F\, d\xi_t\,, \;\; t \geq 0$

$dy_t = Hx_t\, dt + G\, dw_t\,, \;\; y_0 = 0$ (common measurement)

$u_t = \gamma_t\, (y_0^t)$ $\qquad$ $v_t = \mu_t\, (y_0^t)$

$PI = E\{ \int_0^{t_f} [\, |x_t|_Q^2 + |u_t|^2 - |v_t|^2\,]\, dt + |x_{tf}|_{Qf}^2 \}$

$\min_\gamma \max_\mu \; J(\gamma, \mu) \;\rightarrow\; (\gamma^*, \mu^*)$

Still, there exists a common compensator, and restricted CE/separation holds -- but not complete

NZSSDG with common measurements

$dx_t = (Ax_t + Bu_t + Dv_t)\, dt + F\, d\xi_t\,,\ \ t \geq 0$

$dy_t = Hx_t\, dt + G\, dw_t\,,\ \ y_0 = 0$ (common measurement)

$u_t = \gamma_t\,(y_0^{\,t})$ $\qquad v_t = \mu_t\,(y_0^{\,t})$

$PIi = E\{ \int_0^{t_f} [\ |x_t|_{Qi}^2 + |u_t|_{Ri}^2 + |v_t|_{Mi}^2\ ]\, dt + |x_{tf}|_{Qfi}^2 \}$

➔ $J_i(\gamma, \mu)$ ➔ Nash eqm (γ^*, μ^*)

NZSSDG with common measurements

$dx_t = (Ax_t + Bu_t + Dv_t)\, dt + F\, d\xi_t\,,\ \ t \geq 0$

$dy_t = Hx_t\, dt + G\, dw_t\,,\ \ \ y_0 = 0$ (common measurement)

$u_t = \gamma_t\, (y_0^{\,t})$ $\qquad\qquad$ $v_t = \mu_t\, (y_0^{\,t})$

$PIi = E\{ \int_0^{t_f} [\ |x_t|_{Qi}^2 + |u_t|_{Ri}^2 + |v_t|_{Mi}^2\ ]\, dt + |x_{tf}|_{Qfi}^2 \}$

➔ $J_i(\gamma, \mu)$ ➔ Nash eqm (γ^*, μ^*)

CE/separation does not hold -- NE of deterministic NZSDG cannot be used; not neutral

Recap

- No general theory/approach to non-neutrality
- Not all problems with non-classical information are intractable
- It is not only the information structure but also the structure of the performance index that plays an important role
- With battery limitations and energy conservation in remote control applications, further research on problems with non-classical information is needed